Stochastic Geometry of RIS and NT Networks

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Indo-French Seminar
6G Wireless Networks
Challenges and Opportunities
October 10, 2024

Structure of the presentation

- **Zoom 0: Stochastic Geometry and Wireless Networks**
- **Zoom 1: RIS Enhanced Cellular Networks**
- **Zoom 2: NTN Cellular Networks**
- **Zoom 3: RIS & NTN Networks**

ZOOM 0: POISSON STOCHASTIC GEOMETRY

- **A few basic models:**
  - **Spatial Poisson point process**
  - **Spatial Shot–noise fields : Interference**
  - **Poisson–Voronoi tessellation: "Connection to closest "**
- **[Chiu, Stoyan, Kendall and Mecke 13]**
  Stochastic Geometry and its Applications
- **[Baccelli, Blaszczyszyn, Karray 24]**
  Random Measures, Point Processes, & Stochastic Geometry

STOCHASTIC GEOMETRY & CELLULAR NETWORKS

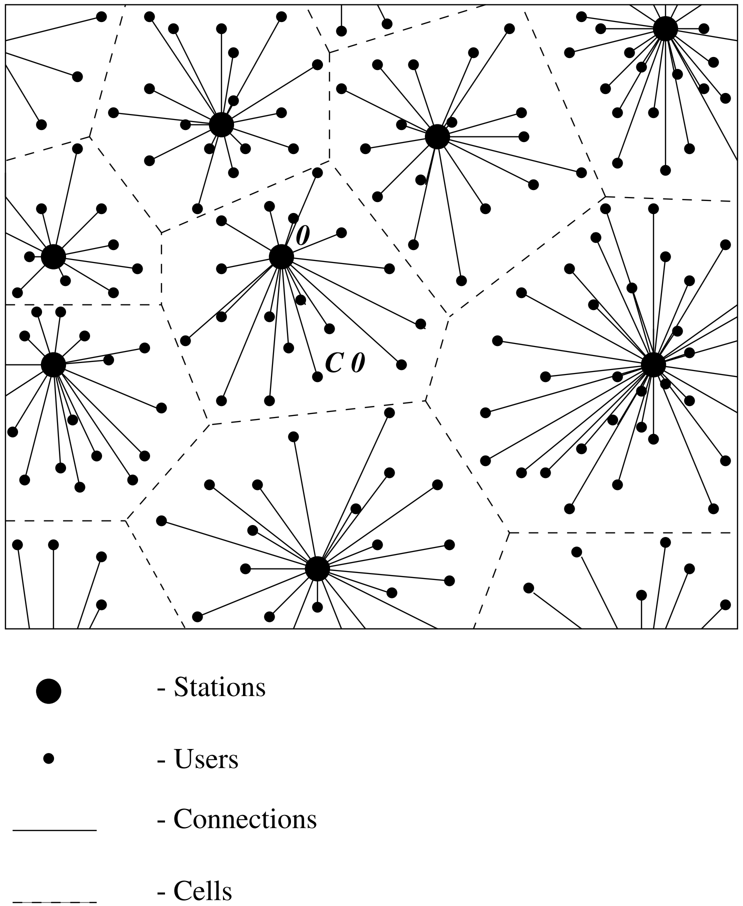
- **Started around 2000**
  branch of stochastic geometry dedicated to wireless networks

- **Extends to system level analysis of other networks**
  WiFi, IoT, NTN

- **Google query** *wireless stochastic geometry* $\rightarrow$ **millions of hits**
  more than 10 books, tens of thousands articles

- **Large scientific community in this field worldwide**

POISSON-VORONOI CELLULAR NETWORKS



- **Base stations** (BSs) arranged according to an homogeneous Poisson point process of intensity λ in $\mathbb{R}^2$
- **UEs**
  - located according to some independent stationary point process
  - each user is served with the closest BS $\rightarrow$ **Poisson Voronoi Cells**

SHANNON RATE IN POISSON-VORONOI CELLULAR NETWORKS WITH RAYLEIGH FADES

- **SINR experienced by typical user**: $\mathbf{SINR} := \frac{\mathbf{S}}{\mathbf{I+N}}$
  - **S: Signal power**: stems from closest BS
  - **I: Interference power**: from BSs outside Voronoi cell of typical user
  - **N**: thermal noise power
- **Shannon rate offered to typical user**: $\mathcal{T} \sim \mathbf{B}\log(\mathbf{1} + \mathbf{SINR})$
- **Question**: Law of the Shannon rate offered to typical user

COVERAGE/SHANNON RATE

$$\mathbf{p_c}(\mathbf{T}, \lambda, \beta) = \Pr_{\mathbf{u}}^{\mathbf{0}}[\mathbf{SINR} > \mathbf{T}] = \Pr_{\mathbf{u}}^{\mathbf{0}}[\text{Shannon rate} > \mathbf{B} \log(\mathbf{1} + \mathbf{T})]$$

- **As in statistical physics, this is equivalent to spatial averages**
  - **Average fraction of users who achieve SINR at least T**
  - **Average fraction of the network area in "T-coverage"**
- **Assumptions on propagation for next result:**
  - **Power law path loss :**
    at distance r, $\mathbf{l}(\mathbf{r}) = \mathbf{r}^{\beta}, \ \beta > \mathbf{2},$ **path loss exp.**
  - **Rayleigh fading model: Exponential fade with mean** $\frac{1}{\mu}$

P-V-S THEOREM FOR RAYLEIGH FADES

- **Theorem**

  J.G. Andrews, F.B and R.K. Ganti, IEEE Tr. Comm. 11

$$\mathbf{p_c}(\mathbf{T},\lambda,\beta)=\pi\lambda\int_{0}^{\infty}\mathbf{e}^{-\pi\lambda\mathbf{v}(\mathbf{1}+\rho)-\mu\mathbf{T}\mathbf{N}\mathbf{v}^{\beta/2}}\mathrm{d}\mathbf{v}\quad\textbf{with}\quad\rho=\mathbf{T}^{\frac{2}{\beta}}\int_{\mathbf{T}^{\frac{-2}{\beta}}}^{\infty}\frac{\mathbf{1}}{\mathbf{1}+\mathbf{u}^{\beta/2}}\mathrm{d}\mathbf{u}$$

  - **Step 1 : Poisson–Voronoi : distance to closest BS: Rayleigh distr. $\rightarrow$ S**
  - **Step 2 : Poisson–Voronoi : Poisson shot-noise outside a ball $\rightarrow$ I**
  - **Step 3 : Poisson–Voronoi–Shannon : law of SINR and Shannon rate**

- **Closed form expressions in e.g. interference limited case**

PROOF

- **Step 1:** **Poisson-Voronoi**

$$\begin{aligned}\mathbf{p_c}(\mathbf{T},\lambda,\beta) &= \mathbb{P}[\mathbf{SINR} > \mathbf{T}] \\ &= \int_{\mathbf{r}>0} \mathbb{P}[\mathbf{SINR} > \mathbf{T}]\mathbf{f_r}(\mathbf{r})\mathrm{d}\mathbf{r} \\ &= \int_{\mathbf{r}>0} \mathbb{P}\left[\frac{\mathbf{S}\mathbf{r}^{-\beta}}{\mathbf{N}+\mathbf{I_r}} > \mathbf{T}\right] \mathbf{e}^{-\pi\lambda\mathbf{r}^2} \mathbf{2}\pi\lambda\mathbf{r}\mathrm{d}\mathbf{r} \\ &= \int_{\mathbf{r}>0} \mathbf{e}^{-\pi\lambda\mathbf{r}^2}\mathbb{P}[\mathbf{S} > \mathbf{T}\mathbf{r}^{\beta}(\mathbf{N}+\mathbf{I_r})]\mathbf{2}\pi\lambda\mathbf{r}\mathrm{d}\mathbf{r}\end{aligned}$$

$\mathbf{I_r}$: interference power given the closest BS is at distance r

PROOF (*continued*)

- **Step 2: Rayleigh. Since $\mathbf{S} \sim \exp(\mu)$,**

$$\mathbb{P}(\mathbf{S} > \mathbf{Tr}^{\beta}(\mathbf{N} + \mathbf{I_r})) = \mathbb{E}[\exp(-\mu\mathbf{Tr}^{\beta}(\mathbf{N} + \mathbf{I_r})] = \mathbf{e}^{-\mu\mathbf{Tr}^{\beta}\mathbf{N}}\mathcal{L}_{\mathbf{I_r}}(\mu\mathbf{Tr}^{\beta})$$

with $\mathcal{L}_{\mathbf{I_r}}(\mathbf{s})$ the Laplace transform of the interference
Thus

$$\mathbf{p_c}(\mathbf{T}, \lambda, \beta) = \int_{\mathbf{r}>0} \mathbf{e}^{-\pi\lambda\mathbf{r}^2}\mathbf{e}^{-\mu\mathbf{Tr}^{\beta}\mathbf{N}}\mathcal{L}_{\mathbf{I_r}}(\mu\mathbf{Tr}^{\beta})\mathbf{2}\pi\lambda\mathbf{r}\mathrm{d}\mathbf{r}$$

PROOF (*continued*)

- **Step 3: Interference as shot noise field**

$$\mathcal{L}_{\mathbf{I_r}}(\mathbf{s}) = \exp\left(-\mathbf{2}\pi\lambda \int_{\mathbf{r}}^{\infty} \left(\mathbf{1} - \mathcal{L}_{\mathbf{F}}\left(\mathbf{s}\mathbf{v}^{-\beta}\right)\right) \mathbf{v}\mathrm{d}\mathbf{v}\right)$$

with $\mathcal{L}_F$ the Laplace Transform of the general fading

PROOF (*continued*)

- **Computational step 4: plugging in $\mathbf{s} = \mu\mathbf{T}\mathbf{r}^{\beta}$ gives**

$$\mathcal{L}_{\mathbf{I_r}}(\mu\mathbf{T}\mathbf{r}^{\beta}) = \exp\left(-\mathbf{2}\pi\lambda\int_{\mathbf{r}}^{\infty}\left(\mathbf{1}-\mathcal{L}_{\mathbf{F}}\left(\mu\mathbf{T}\mathbf{r}^{\beta}\mathbf{v}^{-\beta}\right)\right)\mathbf{v}\mathrm{d}\mathbf{v}\right)$$

$$= \exp\left(-\mathbf{2}\pi\lambda\int_{\mathbf{0}}^{\infty}\left(\int_{\mathbf{r}}^{\infty}(\mathbf{1}-\mathbf{e}^{-\mu\mathbf{T}\mathbf{r}^{\beta}\mathbf{v}^{-\beta}\mathbf{g}})\mathbf{v}\mathrm{d}\mathbf{v}\right)\mathbf{f}(\mathbf{g})\mathrm{d}\mathbf{g}\right)$$

$$= \exp\left(\lambda\pi\mathbf{r}^{\mathbf{2}} - \frac{\mathbf{2}\pi\lambda(\mu\mathbf{T})^{\frac{\mathbf{2}}{\beta}}\mathbf{r}^{\mathbf{2}}}{\beta}\right.$$
$$\left.\int_{\mathbf{0}}^{\infty}\mathbf{g}^{\frac{\mathbf{2}}{\beta}}\left[\mathbf{\Gamma}(-\mathbf{2}/\beta,\mu\mathbf{T}\mathbf{g})-\mathbf{\Gamma}(-\mathbf{2}/\beta)\right]\mathbf{f}(\mathbf{g})\mathrm{d}\mathbf{g}\right)$$

with $\mathbf{f}(\mathbf{g})$ the PDF of $\mathbf{F}$

EXAMPLE OF USE

- **Example of System Level Question that can be answered:**

When does BS densification lead to a decrease of spectral efficiency?

- **When $N = 0$**
  - **Constant spectral efficiency for all densities !**

$$p_c(T, \lambda, \beta) = \frac{1}{1 + \rho(T, \beta)}$$

Scale invariance: only true for power law attenuation

  - **For bounded attenuation functions**

$$p_c(T, \lambda, \beta) \to 0 \text{ as } \lambda \to \infty$$

Joint work with A. Alammouri, J. G. Andrews, IEEE Trans. Information Theory, 2019

STATE OF THE ART – EXTENSIONS – FUTURE

- **Further point processes**: beyond Poisson,

  Joint work w. J.G. Andrews, H. Dhillon, Y. Li, IEEE Tr. Comm. 15

- **Further propagation models**: beyond scale invariance,

  Joint work w. A. Alammouri, J.G. Andrews, IEEE Tr. Inf. Theory 19

- **Obstacle/Shadowing models**: essential for millimeter waves,

  Joint work w. J. Lee, IEEE Infocom 18

- **MIMO and BeamForming** : optimal beam management

  Joint work w. S. Kalamkar and NBL, IEEE Tr. Wireless 22

- and also **Power Control, OFDM, Coexistence with WiFi, Successive Interference Cancellation, CoMP, Vehicular networks, etc.**

STATE OF THE ART – EXTENSIONS – FUTURE (*continued*)

- **Stochastic Geometry for 6G and Beyond**
  - **RIS**
  - **NTN**
  - **Deterministic Latency Networking**
  - **JCAS**
  - **Cell Free**
- **Worldwide efforts engaged on the matter**

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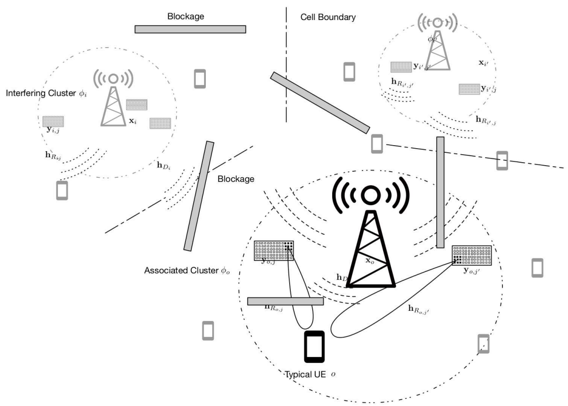
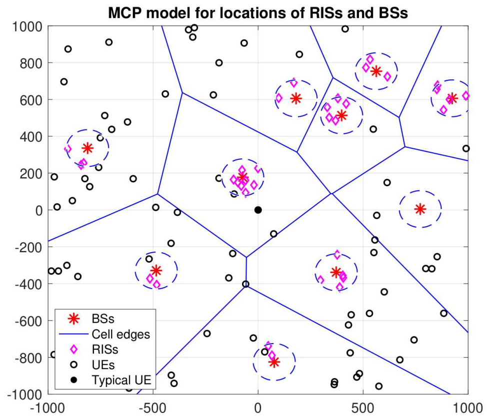
ZOOM 1: RIS ENHANCED CELLULAR NETWORK





RIS cluster with each BS

Blockages BSs → UEs

Each RIS

- **reflects BS signal**
- **beamforms to UEs**

Matérn Cluster Process model

- **BS PPP of intensity λ_{BS}**
- **RIS PPP of intensity λ_{RIS} in ring $[r, R]$ around each BS**
- **UE PPP of intensity λ_{UE}**

MAIN TECHNICAL NOVELTY

- Under **OFDM Assumptions**, Signal has two components
  - Direct path with power Q_{S_D}
  - Reflected paths with RIS beamforming with power Q_{S_R} characterized by its Laplace Transform
- Interference power Q_I characterized by its Laplace Transform (LT of MCP known in closed form)
- **Coverage Probability**

$$\mathrm{P}_c(T) \triangleq \mathbb{P}(\mathrm{SIR} \geq T) = \mathbb{E}_r[\mathbb{P}(\mathrm{SIR} \geq T|r)] = \mathbb{E}_r\left[\mathbb{P}\left(\frac{Q_{S_D}(r) + Q_{S_R}(r)}{Q_I(r)} \geq T\middle|r\right)\right].$$

$$\mathbb{P}\left[\frac{Q_{S_D}(r) + Q_{S_R}(r)}{Q_I(r)} \geq T\middle|r\right] = \mathbb{P}\big[Q_{S_D}(r) \geq TQ_I(r) - Q_{S_R}(r)|r\big].$$

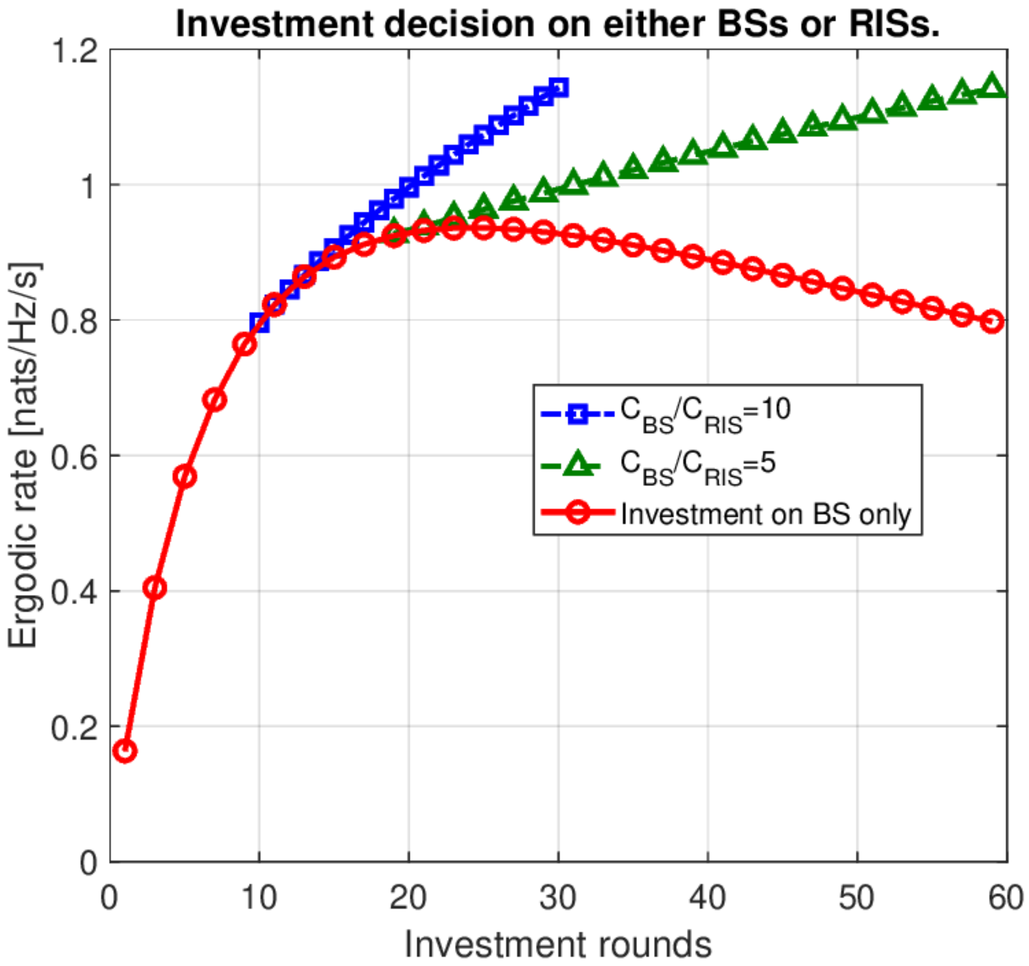
- Need to separate the positive and negative parts in the RHS of the last equation. **Wiener Hopf Factorization**

SYSTEM LEVEL QUESTIONS

- **Influence on Spectral Efficiency of**
  - **Geometry of clusters : more or less spread?**
  - **RIS resources organization**
    Bigger and lesser RISs or the other way around?
- **For optimal configuration**
  - **Mean spectral efficiency gain brought by RISs**
  - **Dependence of this gain in function of obstacle density**

Performance Analysis of RIS-assisted MIMO-OFDM Cellular Networks Based on Matern Cluster Processes, G. Sun, F. Baccelli, K. Feng, L.G. Uzeda Garcia, S. Paris, ArXiv 2024

EXAMPLE OF ECONOMIC ANALYSIS



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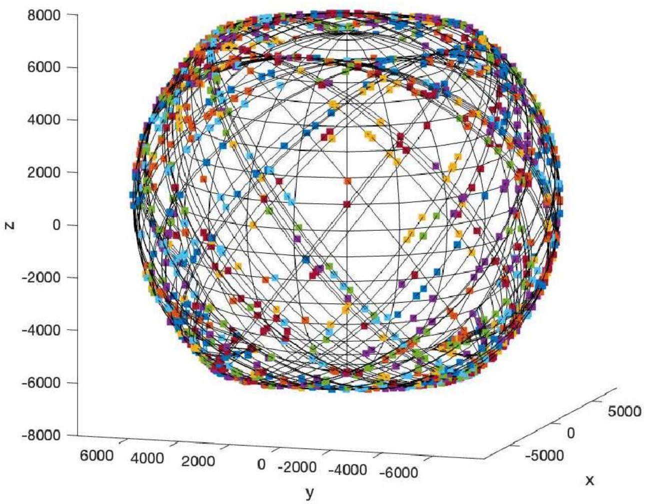
ZOOM 2: NTN BASED CELLULAR NETWORKS



A constellation of LEO satellites

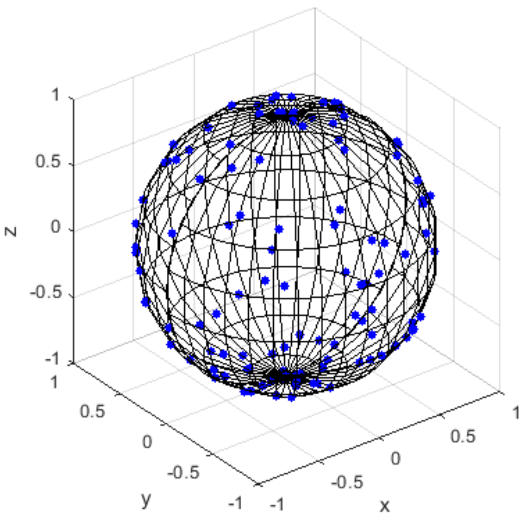
- Spherical geometry with orbiting BSs
- BSs move on orbits with given inclination

- **Need for new SG models**

Orbits?, Voronoi?
Coverage? SINR?
Spectral Efficiency?

- **System level questions**
  - Interaction/Interference between 5G and NTN
  - Optimal orbit/satellite density
  - 5G offloading analysis

FIRST STEPS



A Binomial PP on the sphere [Okati et al. 20] IEEE Trans. Comm.

Binomial p.p. with satellites uniformly distributed on a sphere

Basic questions similar to those on the plane but in spherical geometry

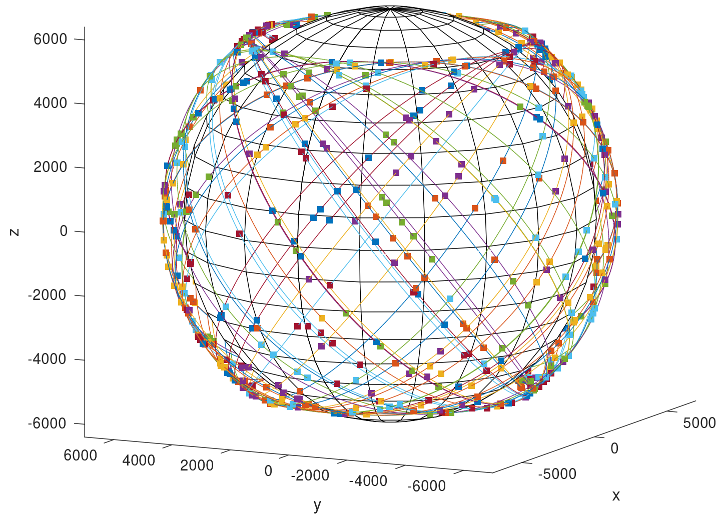
Limitations

1. Geometry: No orbital planes
2. Analysis: clustering of interference ignored

ONGOING STEPS 1

Build a stochastic geometry framework with requirements:

- Characterize orbital planes with various longitudes and inclinations
- Address distribution of LEO satellites on orbital planes
- Evaluate the SINR distribution and spectral efficiency



Cox Point Processes for Multi-Altitude LEO Satellite Networks, C.S. Choi, F. Baccelli, IEEE Trans. Veh. Technol., 2024

ONGOING STEPS 2

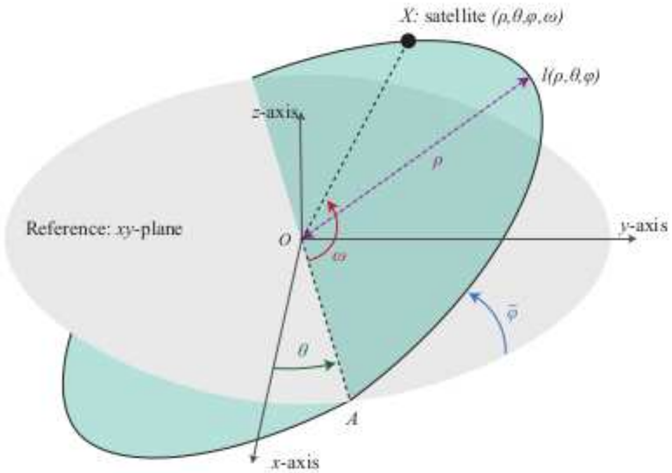
Build a stochastic geometry framework allowing one to:

- **Evaluate impact of NTN on terrestrial**
- **Evaluate synergy between NTN and terrestrial**

Ongoing work with J. Park and N. Lee ArXiv 2024

MAIN TECHNICAL NOVELTIES

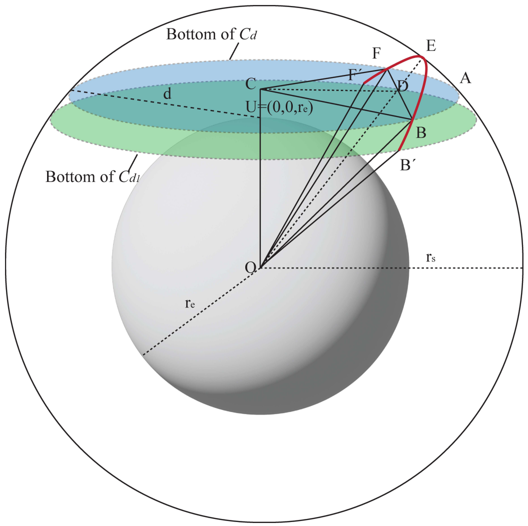
- **Representation of Orbits in SG**



Inclination and Ascending Point

MAIN TECHNICAL NOVELTIES (*continued*)

■ Spherical Geometry



Visibility Cap, Visibility Arc of an Orbit

MAIN TECHNICAL NOVELTIES (*continued*)

- **Distributions Needed to Analyze Downlink SINR**
  - **Signal: Distance to closest visible satellite**
  - **Interference: Shot Noise created by other visible satellites**
- **Extension of the Laplace Transform approach to evaluate**
  - **Probability of Coverage**
  - **Spectral Efficiency**

Latitude dependent for most deployments

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ZOOM 3: NTN AND RIS

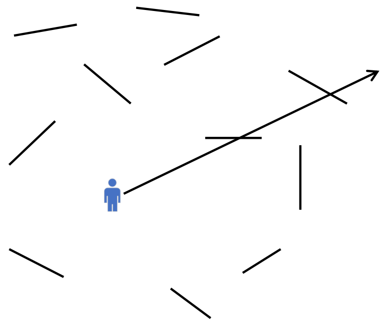
Question: How Much Can Reconfigurable Intelligent Surfaces Augment Sky Visibility ?

- **Urban environments**
- **Millimeter wave bands (either 5G or NTN) blocked by buildings**
- **Connectivity of terrestrial users to NTN entities**
- **Visibility and coverage extension provided by RIS installed on top of buildings**

- **Distribution of Visibility Angle**
- **Distribution of RIS Augmented Visibility Angle**
- **Metrics:**
  – **Angular**
  – **Linear**
  – **Coverage**

SYSTEM MODEL

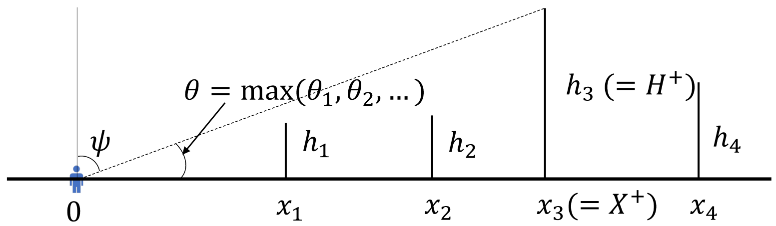
Urban Environment 3D Model



Segments : buildings in a 2D plane

3rd Dimension : building heights

Urban Environment 2D Model



$\{x_i\}$ building locations in chosen direction

$\{h_i\}$ building heights

θ visibility angle

SYSTEM MODEL (*continued*)

- **M/M Model**
  - $\{x_i\}_{i \in \mathbb{Z}}$: **Homogeneous Poisson Point Process** (λ) **on** $\mathbb{R}$
  - $\{h_i\}$ **i.i.d. exponential RVs with parameter** μ
- **M/D Model**
  - $\{x_i\}_{i \in \mathbb{Z}}$: **Homogeneous Poisson Point Process** (λ) **on** $\mathbb{R}$
  - $\{h_i\}$ **constant with common value** μ^{-1}
- **D/M Model ......**
- **Extensions : All other models of queueing theory (e.g. GI/GI.....)**

DISTRIBUTION OF VISIBILITY ANGLE

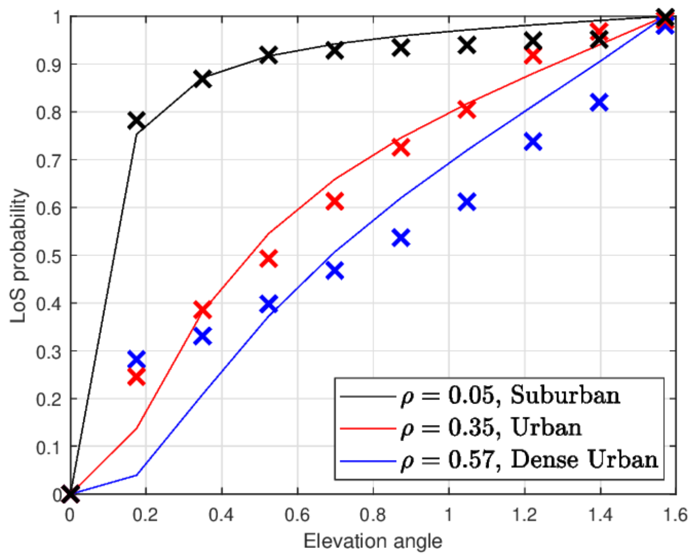
- **THEOREM In the M/M case, the CDF of $\tan\theta$ is**

$$\mathbb{P}[\tan\theta \leq \mathbf{t}] = \mathbf{e}^{-\frac{\rho}{\mathbf{t}}}, \quad \mathbf{t} \geq \mathbf{0},$$

which is a Fréchet distribution with

  - **shape parameter $\alpha = 1$**
  - **scale parameter $\mathbf{s} = \rho = \frac{\lambda}{\mu}$**

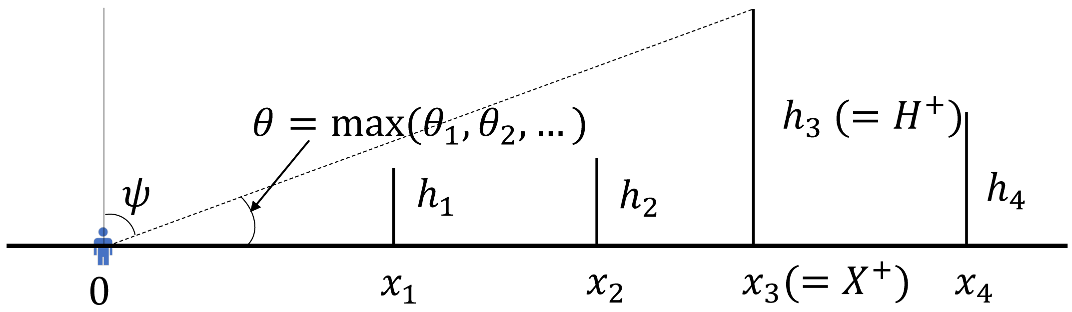
- **Proof obtained from the Laplace Functional of the PPP**



Comparison to 3GPP data

Closed form expressions for M/D and M/W as well

JOINT DISTRIBUTION



- **THEOREM The joint density of (X^+, H^+) at (x, h) is**

$$\mathbf{j(x,h) = \lambda\mu e^{-\mu h - \frac{\lambda x}{\mu h}}, \quad h \geq 0, x \geq 0}$$

JOINT DISTRIBUTION (*continued*)

- **The density of H^+ at h is the Gamma distribution**

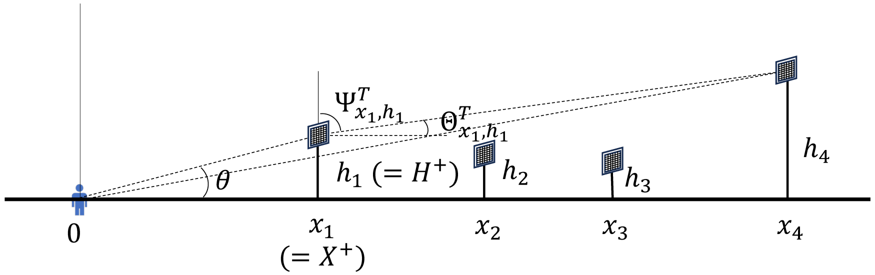
$$g(h) = \mu^2 h e^{-\mu h}$$

- **The density of X^+ at x is**

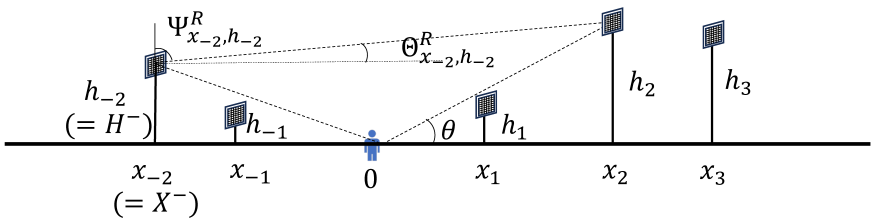
$$k(x) = 2\lambda\sqrt{\lambda x} K_1(2\sqrt{\lambda x})$$

where $K_n(\cdot)$ is the modified Bessel function of the second kind

DISTRIBUTION OF RIS AUGMENTED VISIBILITY ANGLE



Transmissive mode : improved visibility angle $\Theta^{T}_{\mathbf{x},\mathbf{h}}$



Reflective mode : similar definition

DISTRIBUTION

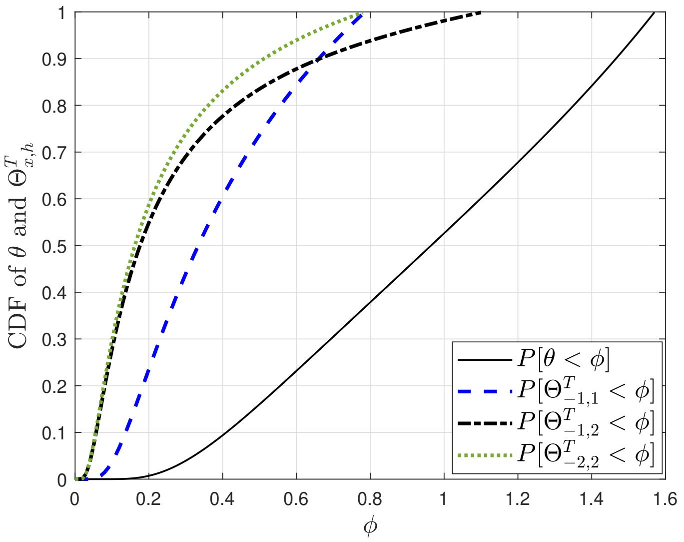
- **THEOREM**

The conditional CDF of $\tan \boldsymbol{\Theta}^{\mathbf{T}}$ **given that** $(\mathbf{X}^{+}, \mathbf{H}^{+}) = (\mathbf{x}, \mathbf{h})$ **is**

$$\mathbb{P}[\tan \boldsymbol{\Theta}^{\mathbf{T}}_{\mathbf{x},\mathbf{h}} \leq \mathbf{t}] = \mathbb{P}[\tan \boldsymbol{\Theta}^{\mathbf{T}} \leq \mathbf{t} | \mathbf{X}^{+} = \mathbf{x}, \mathbf{H}^{+} = \mathbf{h}]$$
$$= \begin{cases} \exp\left(-\rho \left[\frac{\mathbf{1}}{\mathbf{t}} - \frac{\mathbf{x}}{\mathbf{h}}\right] \mathbf{e}^{-\mu \mathbf{h}}\right), & \textbf{for } \mathbf{0} < \mathbf{t} \leq \frac{\mathbf{h}}{\mathbf{x}}, \\ \mathbf{1}, & \textbf{for } \mathbf{t} > \frac{\mathbf{h}}{\mathbf{x}} \end{cases}$$

- **Closed form expressions for conditional**
  - **density**
  - **moments**
- **Closed form expressions for conditional distribution of** $\tan \boldsymbol{\Theta}^{\mathbf{R}}$

DISTRIBUTION (*continued*)



Visibility enhancement by transmissive RISs with $\lambda = 1$ and $\mu = 1$

ANGULAR METRICS

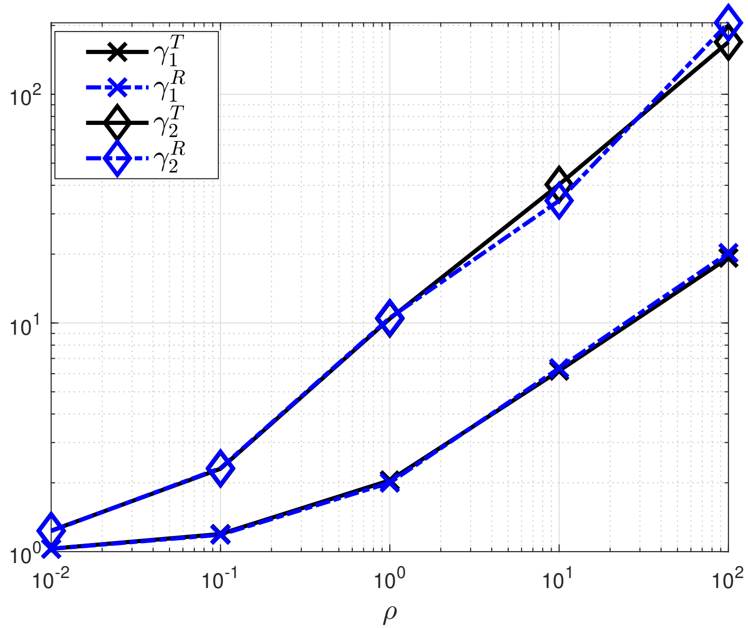
- **Angular metrics** to quantify the visibility enhancement by RISs

$$\gamma_{\mathbf{1}}^{\mathbf{T}} \triangleq \frac{\mathbb{E}[\boldsymbol{\Psi}^{\mathrm{T}}]}{\mathbb{E}[\psi]}, \quad \gamma_{\mathbf{1}}^{\mathbf{R}} \triangleq \frac{\mathbb{E}[\boldsymbol{\Psi}^{\mathrm{R}}]}{\mathbb{E}[\psi]}$$

$$\gamma_{\mathbf{2}}^{\mathbf{T}} \triangleq \mathbb{E}\left[\frac{\boldsymbol{\Psi}^{\mathrm{T}}}{\psi}\right], \gamma_{\mathbf{2}}^{\mathbf{R}} \triangleq \mathbb{E}\left[\frac{\boldsymbol{\Psi}^{\mathrm{R}}}{\psi}\right]$$

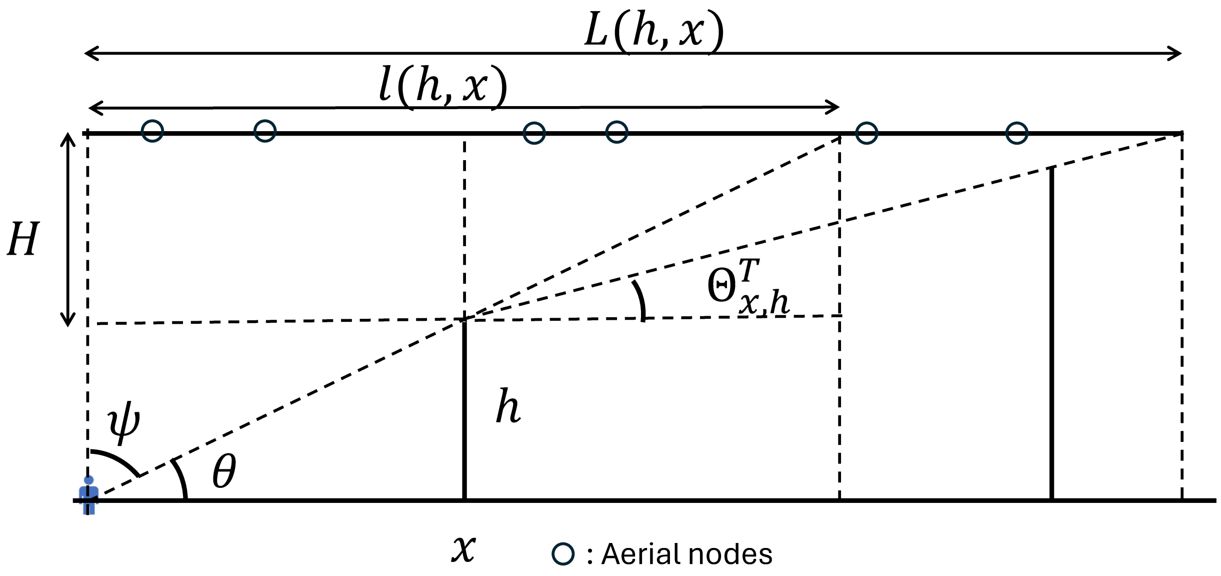
- **Measure how much the visibility angle is increased by using the RISs**
- **Can be evaluated in integral form thanks to the analytical formulas**

ANGULAR METRICS (*continued*)



Visibility enhancement with respect to ρ

LINEAR METRICS



NTN nodes are deployed at the same altitude

Extension of visible regions at this altitude by transmissive RISs

RESULTS ON LINEAR METRICS

- **M/M Model, Transmissive mode**
- **Given** $(\mathbf{X}^+, \mathbf{H}^+) = (\mathbf{x}, \mathbf{h})$

$$|\mathbf{l}(\mathbf{x},\mathbf{h})| = \mathbf{x} + \mathbf{H}\frac{\mathbf{x}}{\mathbf{h}}$$

$$|\mathbf{L}(\mathbf{x},\mathbf{h})| = \mathbf{x} + \frac{\mathbf{H}}{\tan \boldsymbol{\Theta}^{\mathbf{T}}_{\mathbf{x},\mathbf{h}}}$$

- **Conditional Means**

$$|\mathbf{l}(\mathbf{x},\mathbf{h})| = \mathbf{x} + \mathbf{H}\frac{\mathbf{x}}{\mathbf{h}}, \qquad \mathbb{E}[|\mathbf{L}(\mathbf{x},\mathbf{h})|] = \mathbf{x} + \frac{\mathbf{e}^{\mathbf{h}\mu}\mathbf{h} + \mathbf{x}\rho}{\mathbf{h}\rho}\mathbf{H}$$

- **Means**

$$\mathbb{E}[|\mathbf{l}|] = \frac{\mathbf{2} + \mathbf{H}\mu}{\lambda}, \qquad \mathbb{E}[|\mathbf{L}|] = \infty$$

PROBABILITY OF COVERAGE

- **UAVs assumed to be distributed as a homogeneous PPP Φ_u with intensity ν at altitude $h + H$**
- $\tau(x, h)$**: conditional probability of coverage given (x, h) and given no initial coverage**

$$\tau(x,h) = \mathbb{P}[\Phi_u(L(x,h)) > 0 | \Phi_u(\ell(x,h)) = 0] = 1 - \frac{\rho}{e^{h\mu}H\nu + \rho}$$

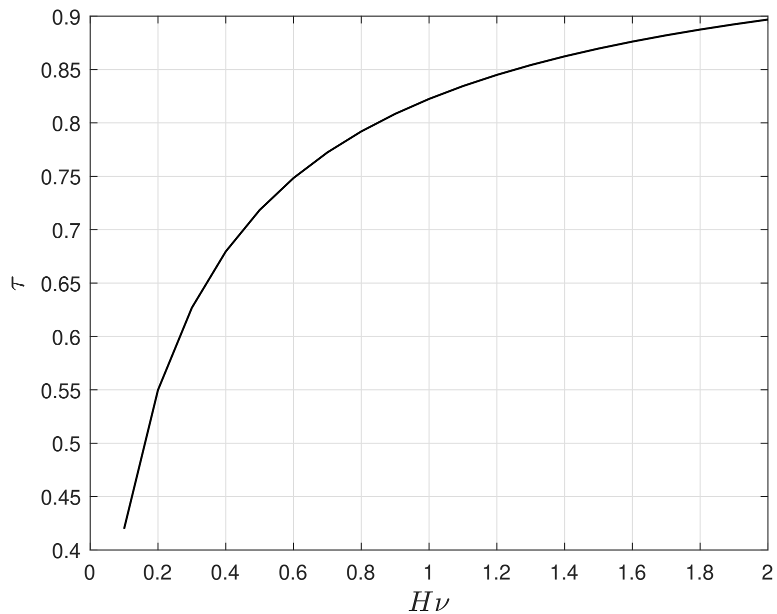
- **Unconditioning**

$$\tau = \frac{H\nu}{6\rho}\left(\pi^2 + 6\log\left(\frac{\rho}{H\nu}\right)\log\left(1+\frac{\rho}{H\nu}\right) - 3\left(\log\left(1+\frac{\rho}{H\nu}\right)\right)^2 - 6\mathrm{Li}_2\left(\frac{H\nu}{H\nu+\rho}\right)\right)$$

where $\mathrm{Li}_n(z)$ is the polylogarithm function

$$\mathrm{Li}_n(z) = \sum_{k=1}^{\infty}\frac{z^k}{k^n}$$

NUMERICAL RESULTS



τ as a function of Hν for $\lambda, \mu = 1$

EXAMPLES: REAL ENVIRONMENTS

	Case 1 (Dense urban)	**Case 2 (Urban)**	**Case 3 (Suburban)**
$\lambda(m^{-1})$	0.012	0.007	0.001
$\mu(m^{-1})$	0.02	0.02	0.02
$\mathbb{E}[\theta]$ (rad)	0.7732	0.5935	0.1695
$\mathbb{E}[\theta^T]$ (rad)	0.1956	0.1256	0.0195
$\mathbb{E}[\theta^R]$ (rad)	0.2214	0.1453	0.0201
$\mathbb{E}[l]$ (HAP) (m)	1.68×10^4	2.89×10^4	2.02×10^5
$\mathbb{E}[l]$ (Satellite) (m)	8.34×10^5	1.43×10^6	1.00×10^7
τ_H (HAP)	0.797838	0.864512	0.976052
τ_H (Satellite)	0.893742	0.935147	0.989409

Numerical values of the visibility for two cities

ONGOING/FUTURE RESEARCH

- **3D**
- **Multi Hop**
- **Refined statistics**
- **SINR coverage**
- **Economic model**

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