Multi-user Communication using OTFS

Talk at Indo-French Seminar

“6G Wireless Networks: Challenges and Opportunities”

Kuntal Deka
Assistant Professor, Department of EEE, IIT Guwahati

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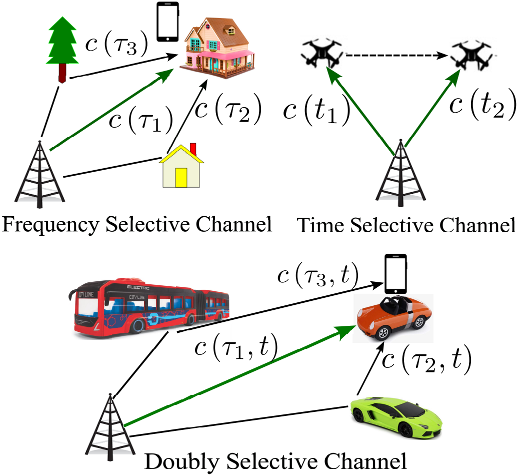
Outline

- Wireless Channel in Delay-Doppler Domain
- Information Symbols over Delay-Doppler Domain
- Orthogonal Time Frequency Space (OTFS)
- Sparse Code Multiple Access (SCMA)
- OTFS-SCMA
- Convolutional Sparse Coding based Channel Estimation of OTFS-SCMA

Wireless Channels

- A wireless channel can be represented in terms of impulse responses.
- If the BS, MS, and IOs are all static, then the channel is time invariant, with an impulse response $c(\tau)$; LTI system
- For time-varying channel, the impulse response is denoted by $c(\tau, t)$; LTV system:

$$y(t) = \int_{-\infty}^{\infty} x\left(t-\tau\right) c\left(\tau, t\right) d\tau$$



Frequency Selective Channel

Time Selective Channel

Doubly Selective Channel

System-Theoretic Description of Wireless Channels

- $c(\tau, t)$, depends on two variables, τ and t, we can perform Fourier transformations with respect to either (or both) of them. This results in four different, but equivalent, representations.
- Fourier transforming the impulse response with respect to the variable τ results in the time-variant transfer function $H(t, f)$:

$$H\left(t, f\right) = \int\limits_{-\infty}^{\infty} c\left(\tau, t\right) \exp\left(-j2\pi f\tau\right) d\tau$$

- A Fourier transformation of the impulse response with respect to t results in delay-Doppler response (Doppler-variant impulse response) $h(\tau, \nu)$:

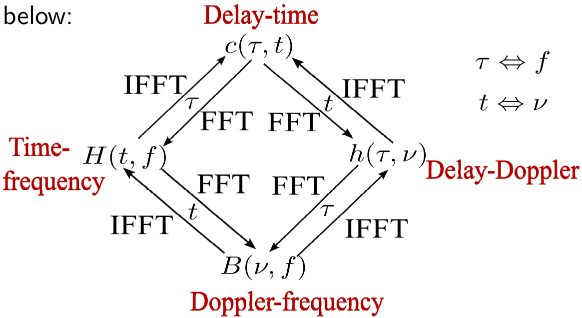
$$h\left(\tau, \nu\right) = \int\limits_{-\infty}^{\infty} c\left(\tau, t\right) \exp\left(-j2\pi\nu t\right) dt$$

System-Theoretic Description of Wireless Channels *contd..*

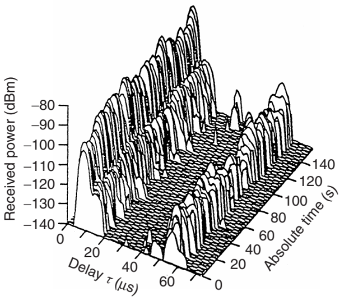
- Finally, the function $h(\tau,\nu)$ can be transformed with respect to the variable τ, resulting in the Doppler-variant transfer function $B(\nu,f)$:

$$B\left(\nu,f\right)=\int\limits_{-\infty}^{\infty}h\left(\tau,\nu\right)\exp\left(-j2\pi f\tau\right)d\tau$$

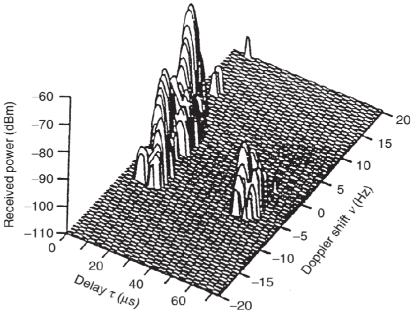
- A summary of the interrelations between the system functions is given below:



Wireless Channel in delay-Doppler Domain [1]



delay-time



delay-Doppler **(Sparse)**

- Special features: Sparsity, Separability, and Stability.

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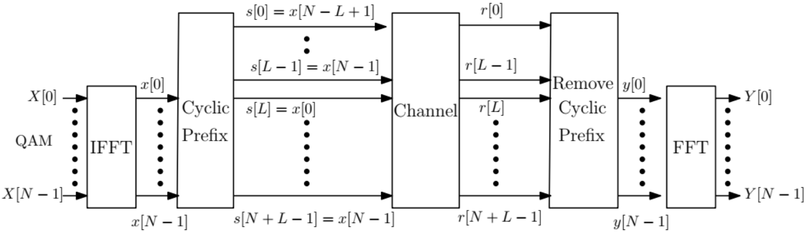
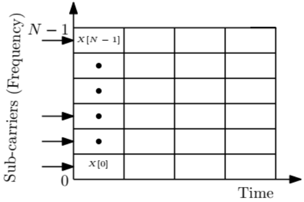
[1] *Wireless Communication*, by Andrew Molisch, John Wiley & Sons, 2012.

OFDM

Orthogonal Frequency Division Multiplexing (OFDM)



- OFDM inserts the data symbols in the (time)-frequency domain.

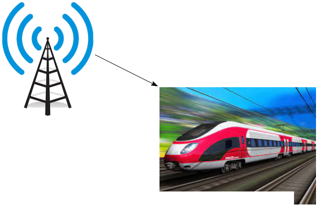


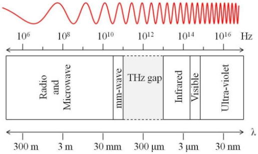
Motivations

- High Doppler will be a major problem for 6G communication.
- Doppler shift is given by

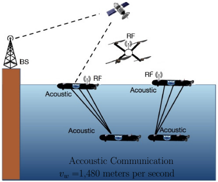
$$f_D \propto \frac{v f_c}{v_w}$$

  where $v \rightarrow$ speed of UE, $f_c \rightarrow$ carrier frequency, and v_w is the speed of the propagation wave.
- Causes of Doppler:
  - High speed vehicles (v) like bullet trains.
  - High carrier frequency (f_c) like mmWave/THz communication.
  - Slow propagation medium (v_w) like acoustic waves in underwater communication ($v_w = 1,480$ m/s).
- Orthogonal Time Frequency Space (OTFS) modulation.





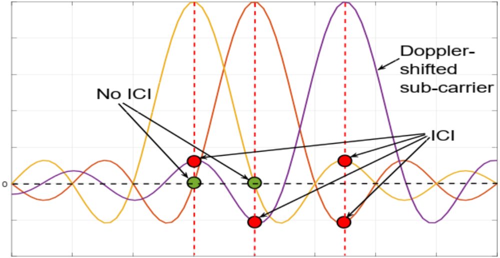
300 GHz–10 THz



High Doppler in OFDM

OFDM in High Doppler Situation

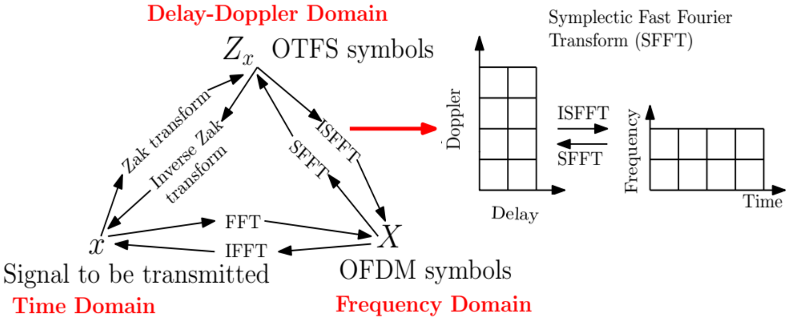
- Doppler shift is given by
$$f_D \propto \frac{v f_c}{v_w}$$
where $v \rightarrow$ speed of UE, $f_c \rightarrow$ carrier frequency, and v_w is the speed of the propagation wave.
- OFDM is sensitive to high Doppler.
- Inter-carrier interference (ICI).



- What if we insert the data symbols in the delay-Doppler domain?

Delay-Doppler Domain

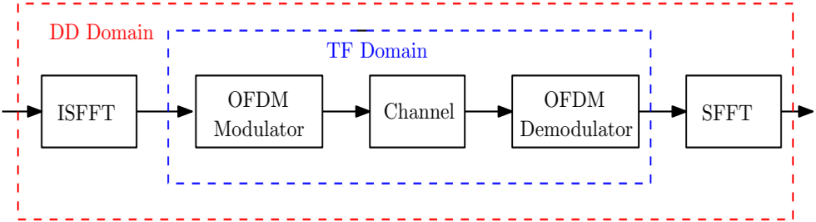
Delay-Doppler Domain



- A time-domain signal may be time limited or frequency limited but not both simultaneously.
- There exists a set of time-domain signals localized simultaneously in delay and Doppler, which can be used as basis signals to devise OTFS modulation.

OTFS [2]

Orthogonal Time Frequency Space (OTFS) Modulation



- OTFS is compatible with existing OFDM systems.
- Why is OTFS better than OFDM?
- The fraction of interfered symbols is less in OTFS compared to OFDM.

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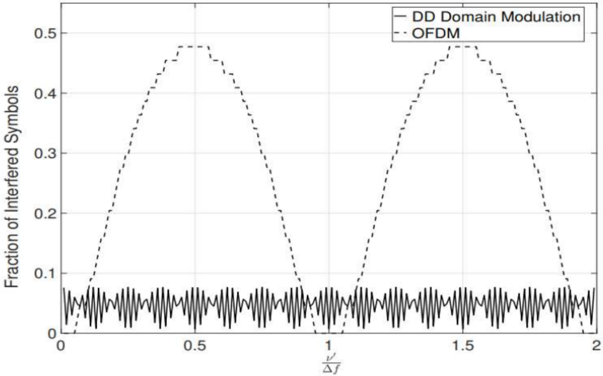
[2] R. Hadani et al., "Orthogonal Time Frequency Space Modulation," 2017 IEEE Wireless Communications and Networking Conference (WCNC), San Francisco, CA, USA, 2017, pp. 1-6, doi: 10.1109/WCNC.2017.7925924.

OTFS vs. OFDM [3]

Why does OTFS perform better than OFDM in high Doppler?



Fraction of interfered information symbols vs. $\frac{\nu'}{\Delta f}$ for CP-OFDM and DD domain modulation.

- The fraction of interfered symbols is less in OTFS compared to OFDM.

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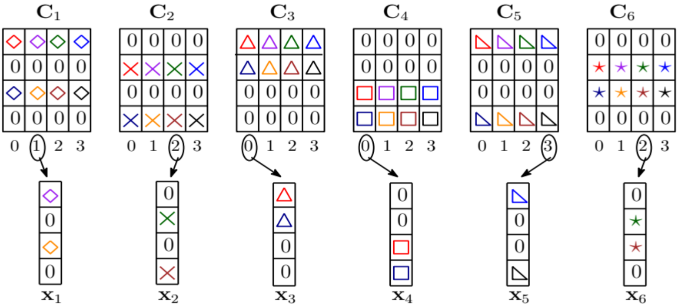
[3] S. K. Mohammed, "Derivation of OTFS Modulation From First Principles," in IEEE Transactions on Vehicular Technology, vol. 70, no. 8, pp. 7619-7636, Aug. 2021, doi: 10.1109/TVT.2021.3069913.

Sparse Code Multiple Access (SCMA)

Sparse Code Multiple Access (SCMA)



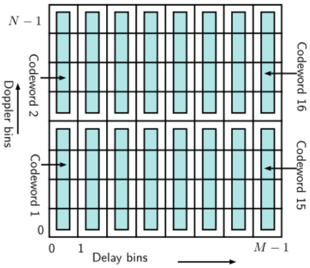
- SCMA: code-domain NOMA technique.
- Incoming data streams are directly mapped to multi-dimensional codebooks.
- Shaping gain.
- SCMA codebook has a critical impact on the performance.

> - Dedicated multi-dimensional sparse codebooks.
> - $J \times K$ SCMA system, $K < J$.
> - Nonzero elements $N < K$ in each column.

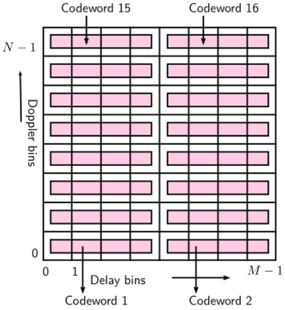
OTFS-SCMA [4]

SCMA codeword allocation in DD domain

- A code-domain NOMA approach for OTFS using $J \times K$ SCMA scheme.
- J users access the NM resources simultaneously using sparse codewords $(K \times 1)$.
- The overloading factor is same as of basic SCMA model, $\frac{J}{K}$.



(a) Scheme-1



(b) Scheme-2

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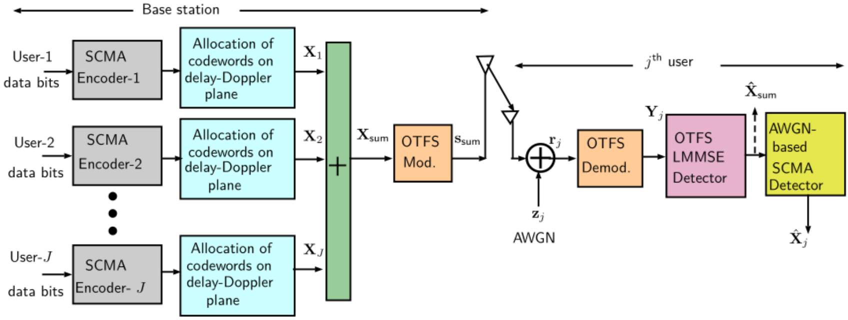
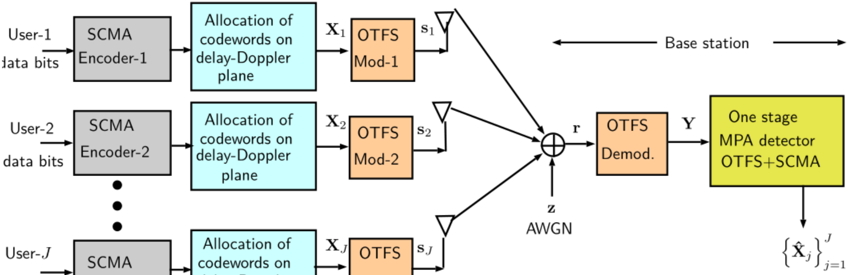
[4] K. Deka, A. Thomas and S. Sharma, "OTFS-SCMA: A Code-Domain NOMA Approach for Orthogonal Time Frequency Space Modulation," in IEEE Transactions on Communications, vol. 69, no. 8, pp. 5043-5058, Aug. 2021, doi: 10.1109/TCOMM.2021.3075237.

OTFS-SCMA in downlink



- $\mathbf{y}_{j,\text{vec}} = \mathbf{H}_j \mathbf{x}_{\text{sum,vec}} + \mathbf{z}_j$
- Message detection
  - OTFS LMMSE detection: $\hat{\mathbf{x}}_{\text{sum,vec}} = \mathbf{H}^{\dagger}{}_j [\mathbf{H}_j \mathbf{H}^{\dagger}{}_j + \sigma^2{}_n \mathbf{I}_{MN}]^{-1} \mathbf{y}_{j,\text{vec}}$
  - AWGN based SCMA detection

OTFS-SCMA in uplink



$$\mathbf{y}_{\text{vec}} = \sum_{j=1}^{J} \mathbf{H}_j \mathbf{x}_{j,\text{vec}} + \mathbf{z}$$

$$= \mathbf{H}_{\text{all}} \mathbf{x}_{\text{all}} + \mathbf{z}$$

OTFS and SCMA modulation effects are inseparable from $\mathbf{H}_{\text{all}}$.

where $\mathbf{H}_{\text{all}} = [\mathbf{H}_1 \mathbf{H}_2 \ldots \mathbf{H}_J]$ and $\mathbf{x}_{\text{all}} = [\mathbf{x}_{1,\text{vec}}^T \mathbf{x}_{2,\text{vec}}^T \ldots \mathbf{x}_{J,\text{vec}}^T]^T$

$$\mathbf{y}_{\mathsf{vec}} = \begin{bmatrix} y_1 \\ \vdots \\ y_{MN} \end{bmatrix} = \begin{bmatrix} \mathbf{h}_{1,1} & \cdots & \mathbf{h}_{1,\frac{JMN}{K}} \\ \vdots & \ldots & \vdots \\ \mathbf{h}_{MN,1} & \cdots & \mathbf{h}_{MN,\frac{JMN}{K}} \end{bmatrix} \begin{bmatrix} \mathbf{x}_1 \\ \vdots \\ \mathbf{x}_{\frac{JMN}{K}} \end{bmatrix} + \begin{bmatrix} z_1 \\ \vdots \\ z_{MN} \end{bmatrix} \tag{1}$$

The d_v elements in $\mathbf{x}_j$ should be considered together as a single entity or variable node as they correspond to a particular information symbol in the input side.

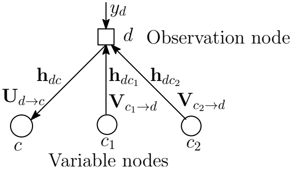


Figure: The update of message from an observation node in the single-stage MPA detection of OTFS-SCMA in uplink.

$$U_{d\to c}(m) = \sum_{(\mathbf{v}_{c_1},\mathbf{v}_{c_2})\in\mathbb{A}_{c_1}\times\mathbb{A}_{c_2}} \frac{1}{\pi N_0}\exp[-\frac{1}{N_0}|y_d - \mathbf{h}_{dc}\mathbf{x}_{cm} - \mathbf{h}_{dc_1}\mathbf{v}_{c_1} - \mathbf{h}_{dc_2}\mathbf{v}_{c_2}|^2]V_{c_1\to d}(\mathbf{v}_{c_1})V_{c_2\to d}(\mathbf{v}_{c_2})$$

Diversity Analysis of OTFS-SCMA

Theorem

Consider an OTFS-SCMA system with an $N \times M$ *delay-Doppler grid* Γ_{NM} *and a* $J \times K$ *SCMA model with* N *and* M *being integer multiples of* K. *Let the wireless channel for the* j^{th} *user be represented by* P *multipaths with the integer delay-Doppler tap pairs* $\left(l^j_{\tau_i}, k^j_{\nu_i}\right)$, $i = 1, \cdots, P, j = 1, \ldots, J$. *Consider the sets* $S^j_k = \left\{\left[k^j_{\nu_1}\right]_K, \left[k^j_{\nu_2}\right]_K, \ldots, \left[k^j_{\nu_P}\right]_K\right\}$ *and* $S^j_l = \left\{\left[l^j_{\tau_1}\right]_K, \left[l^j_{\tau_2}\right]_K, \ldots, \left[l^j_{\tau_P}\right]_K\right\}$. *In the downlink, for the* j^{th} *user, the asymptotic diversity orders for Scheme-1 and Scheme-2 are given by* $\left|S^j_k\right|$ *and* $\left|S^j_l\right|$, *respectively. In the uplink, the asymptotic diversity orders for Scheme-1 and Scheme-2 are given by* $\min\left\{\left|S^1_k\right|, \left|S^2_k\right| \ldots, \left|S^J_k\right|\right\}$ *and* $\min\left\{\left|S^1_l\right|, \left|S^2_l\right| \ldots, \left|S^J_l\right|\right\}$, *respectively.*

Diversity Analysis of OTFS-SCMA: Various Schemes

- Scheme-1: Number of distinct mod-K Doppler taps.
- Scheme-2: Number of distinct mod-K delay taps.

Table: Summary of asymptotic diversity of the schemes for $P = 2$, given $(k_{\nu_1}, l_{\tau_1}) = (0, 0)$.

(k_{ν_2}, l_{τ_2})	Scheme-1	Scheme-2
$k_{\nu_2} \neq 0, l_{\tau_2} \neq 0$	2	2
$k_{\nu_2} \neq 0, l_{\tau_2} = 0$	2	1
$k_{\nu_2} = 0, l_{\tau_2} \neq 0$	1	2

Depending on the channel condition, the codeword allocation scheme can be scheduled.

Another Pattern

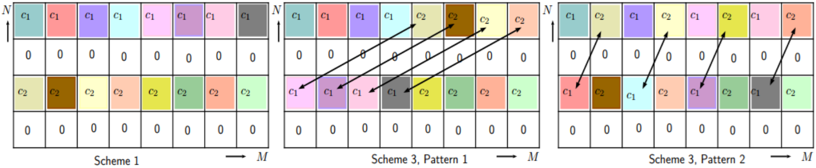


Figure: Two interleaving patterns for *Scheme-3*

Optimal Codeword Allocation Scheme

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Algorithm 1: Optimal codeword allocation scheme
input  : OTFS-SCMA Parameters: $N, M, K$; Channel parameters:
         $\mathcal{P} = \{(k_{\nu_1}, l_{\tau_1}), (k_{\nu_2}, l_{\tau_2}), \ldots, (k_{\nu_P}, l_{\tau_P})\}$
output: Optimal scheme
Initialization: $|S_k|$=Number of distinct mod-$K$ Doppler taps; $|S_l|$=Number of
distinct mod-$K$ delay taps;
if $P \leq K$ then
    if $|S_k| = P$ then
        Scheme-1
    else
        if $|S_l| = P$ then
            Scheme-2
        else
            Scheme-3: Using $\mathcal{P}$, design an interleaving pattern such that
            asymptotic diversity order is $P$
else
    Scheme-3: Using $\mathcal{P}$, design an interleaving pattern such that asymptotic
    diversity order is $P$
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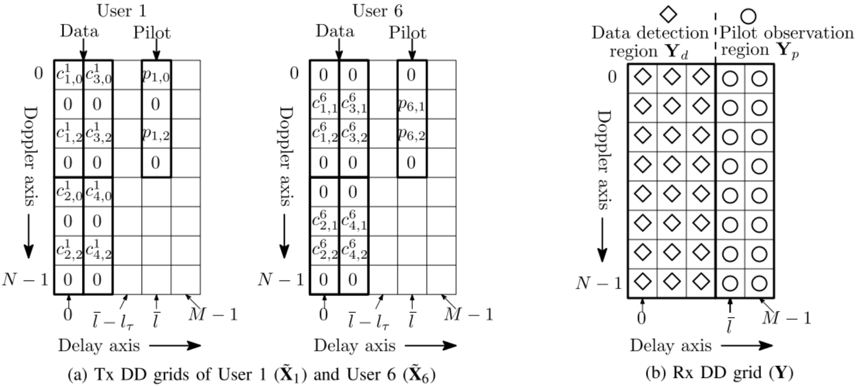
Convolutional Sparse Coding based Channel Estimation [5]



(a) Tx DD grids of User 1 ($\tilde{\mathbf{X}}_1$) and User 6 ($\tilde{\mathbf{X}}_6$)

(b) Rx DD grid ($\mathbf{Y}$)

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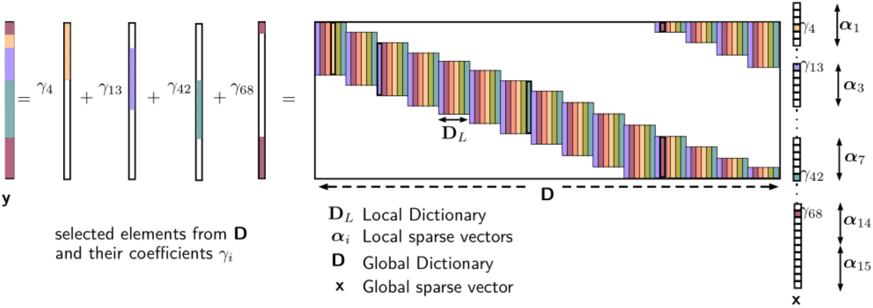
[5] A. Thomas, K. Deka, P. Raviteja and S. Sharma, "Convolutional Sparse Coding Based Channel Estimation for OTFS-SCMA in Uplink," in IEEE Transactions on Communications, vol. 70, no. 8, pp. 5241-5257, Aug. 2022, doi: 10.1109/TCOMM.2022.3182402.

Convolutional Sparse Coding

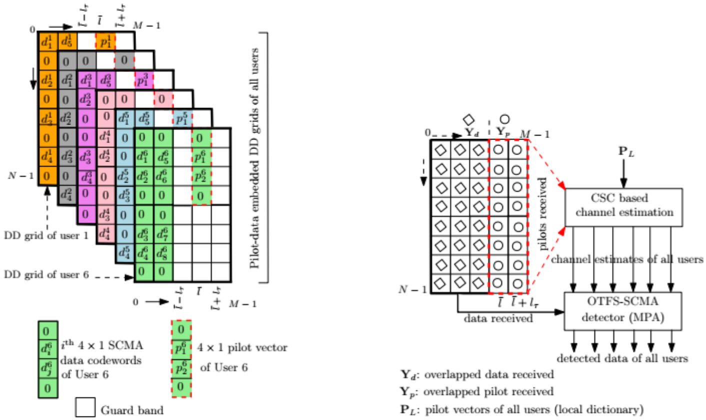


Dictionary structure of convolutional sparse coding.

- OTFS is also a 2D convolution process.
- Channel estimation is a sparse signal recovery problem.

Challenges:
- Formulate channel estimation as CSC problem
  - Obtain dictionary structure from pilot vectors

Channel Estimation and Detection Procedure



(a) Transmitted DD grids of all users.

(b) Received DD grid.

Figure: Overview of the proposed channel estimation method for $\Gamma_{N,M}$, $N = 8, M = 5, J = 6, K = 4, l_\tau = 1,$ and $L_p = 4$.

Analysis of the pilot vector length

Lemma: 1 For successful channel estimation using the proposed method, the length L_p of pilot vector must satisfy the following condition:

$$L_p \geq \max\left\{2J, \lceil cJ\log(J(2k_\nu+1))\rceil - 2k_\nu, k_\nu+1\right\} \quad \text{with } [L_p]_K = 0$$

- For EVA channel model:
  $J = 6,\ K = 4,\ k_\nu = 16,\ c = 1.2$

$$\mathbf{L_p \geq 20}$$

Ongoing Works

- Design of OTFS for other multiple access techniques.

- Comparison of variants of OTFS.

Thank You
Questions/Comments??